FINAL EXAMINATION

ECE 627

June 11, 2010, 9:30 – 11:20 am

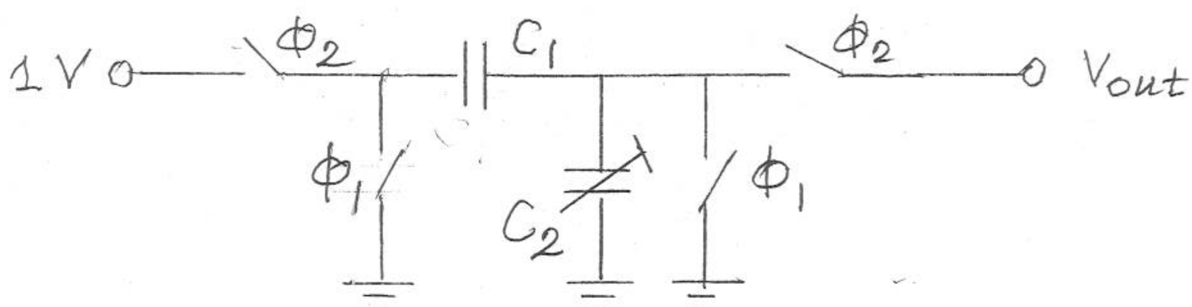
STAG 109

Open book, open notes

1. In the DAC shown, C_2 is controlled by the digital input signal. Both capacitors have uncorrelated random errors, at most 0.1 % in magnitude.

   a. Find the largest possible error in v_{out} as a function of C_1/C_2;

   b. For what value of C_1/C_2 will the error in v_{out} reach its maximum? How large is this error?

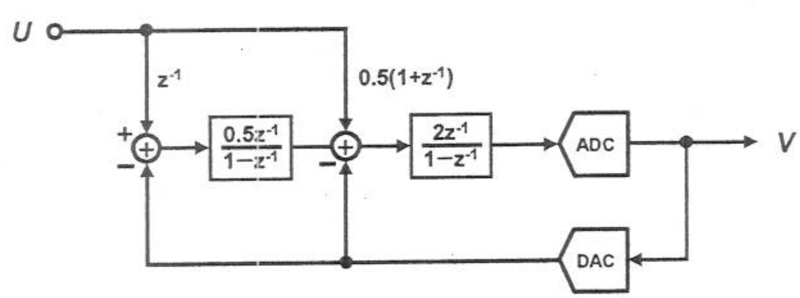


2. In the delta-sigma ADC shown, the quantizer has 17 levels. The reference voltage is 2 V.

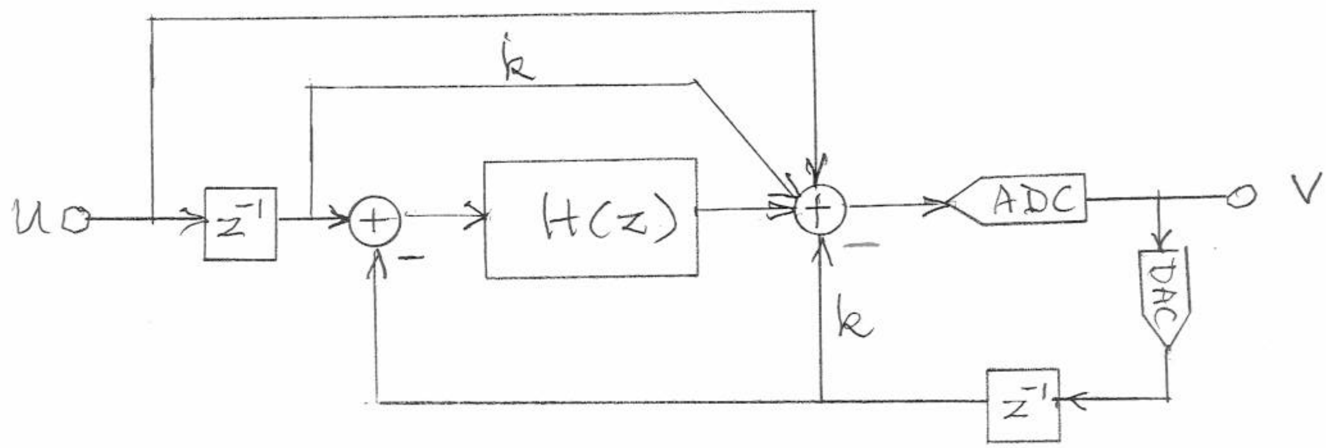
   a. Find the STF and NTF of the ADC;

   b. How large can the output of the first integrator be?

   c. Estimate the largest input voltage for guaranteed absolute stability of the loop.



3. Find the STF of the modulator shown below.



1、1) $V_{out,ideal} = \dfrac{1}{1+\dfrac{C_2}{C_1}}$

assuming C_1 has a random error of α

C_2 has a random error of β

$$V_{out,real} = \frac{1}{1+\frac{C_2(1+\beta)}{C_1(1+\alpha)}} = \frac{1}{1+\theta\frac{C_2}{C_1}}$$

here $\theta = \dfrac{1+\beta}{1+\alpha}$

so $V_{out,error} = V_{out,ideal} - V_{out,real}$

$$= \frac{1}{1+\frac{C_2}{C_1}} - \frac{1}{1+\theta\frac{C_2}{C_1}} = \frac{(\theta-1)\frac{C_2}{C_1}}{(1+\frac{C_2}{C_1})(1+\theta\frac{C_2}{C_1})}$$

$$\approx \frac{(\theta-1)\frac{C_2}{C_1}}{(1+\frac{C_2}{C_1})^2}$$

if $|\alpha| < 0.001$ $|\beta| < 0.001$

$$\Rightarrow \frac{0.999}{1.001} < \theta < \frac{1.001}{0.999} \Rightarrow -\frac{0.002}{1.001} < \theta - 1 < \frac{0.002}{0.999}$$

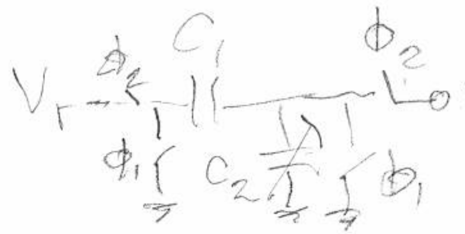
So the largest possible voltage error is

$$\frac{0.002}{0.999}\frac{\frac{C_2}{C_1}}{1+(\frac{C_2}{C_1})^2} = \frac{0.002}{0.999}\frac{\frac{C_1}{C_2}}{1+(\frac{C_1}{C_2})^2}$$

2) $\dfrac{0.002}{0.999}\dfrac{\frac{C_1}{C_2}}{1+(\frac{C_1}{C_2})^2} = \dfrac{0.002}{0.999}\dfrac{1}{\frac{C_1}{C_2}+\frac{C_2}{C_1}+2} \le \dfrac{0.002}{0.999}\cdot\dfrac{1}{4}$

$\cong 5\times10^{-4}$ V

when $\dfrac{C_1}{C_2} = 1$



$V_0 = V_r \frac{C_1}{\underbrace{C_1 + C_2}_{C_t}}$, $\frac{\partial V_0}{\partial C_1} = V_r \frac{\Delta C_2}{C_t^2}$, $\frac{\partial V_0}{\partial C_2} = V_r \frac{-C_1 + C_2 - C_2}{C_t^2}$

$$\Delta V_0 = \frac{V_r}{C_t^2}(-C_1 \Delta C_2 + C_2 \Delta C_1) = V_r \frac{C_1 C_2}{C_t^2}\left(\frac{\Delta C_1}{C_1} - \frac{\Delta C_2}{C_2}\right)$$

$$\frac{\Delta V_0}{V_r} = \underbrace{\frac{C_1 C_2}{C_t^2}}_{a}\left(\frac{\Delta C_1}{C_1} - \frac{\Delta C_2}{C_2}\right), \quad 1/a = \frac{(C_1 + C_2)^2}{C_1 C_2} = \left(\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}}\right)^2 = (r + 1/r)^2$$

$$\frac{\partial a}{\partial r} = -2(r + 1/r)(1 - 1/r^2) \to 0 \qquad r = 1/r = \pm 1 , \quad C_1 = C_2$$

$$V_0 = V_r/2 , \quad a = 1/4 , \quad \left|\frac{\Delta V_0}{V_r}\right|_{max} = \frac{1}{4}\left(\left|\frac{\Delta C_1}{C_1}\right|_{max} + \left|\frac{\Delta C_2}{C_2}\right|_{max}\right) \to \frac{1}{2}\left|\frac{\Delta C}{C}\right|_{max}$$

$$= \frac{10^{-3}}{2}$$

2. 1) $\left\{ (UZ^{-1}-V)\dfrac{0.5Z^{-1}}{1-Z^{-1}} + U\cdot 0.5(1+Z^{-1}) - V \right\} \dfrac{2Z^{-1}}{1-Z^{-1}} + E = V$

$\Rightarrow \quad V = UZ^{-1} + E(1-Z^{-1})^2$

$STF = Z^{-1} \quad NTF = (1-Z^{-1})^2$

2) $V_1 = -\dfrac{1}{2} Z^{-1}(1-Z^{-1})E$ $\qquad LSB = \dfrac{2}{16} V$

$= -\dfrac{1}{2} Z^{-1}E + \dfrac{1}{2} Z^{-2}E$ $\qquad |E| < \dfrac{1}{2} LSB = \dfrac{1}{16} V$

So $\quad V_1\ max = \dfrac{1}{16} V$

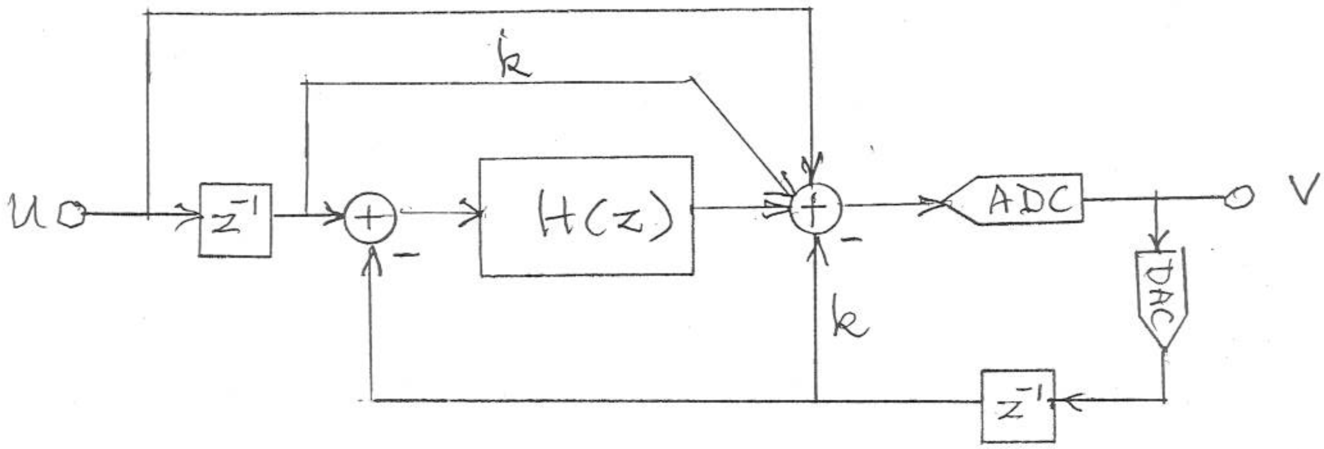
3) $\quad V_2 = Z^{-1}\cdot U + E(Z^{-2} - 2Z^{-1})$

because $V_2 < 2 + \dfrac{1}{2} LSB = \dfrac{33}{16}$

$\left| E(Z^{-2} - 2Z^{-1}) \right| < \dfrac{3}{16}$

$\therefore \quad U_{max} = \dfrac{30}{16} = \dfrac{15}{8} V$

3. Find the STF of the modulator shown below.



$$V\big|_{E=0} = -kz^{-1}V_o + (1+z^{-1}k)U + Hz^{-1}(U - V_o)$$

$$V_o(1 + kz^{-1} + Hz^{-1}) = U(1 + kz^{-1} + Hz^{-1})$$

$$STF = \frac{V_o}{U} \equiv 1$$